EDITOR’S NOTE
Struggling students require optimized strategies and support to improve their math fluency. This Spotlight will help you identify the importance of math fact fluency; evaluate the challenges student face with word problems; explore how teachers can optimize how they introduce math facts and better support students; gain insights into how early math supports help vulnerable students; create targeted supported for algebra 1 students; and more.

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What Is Math 'Fact Fluency,' and How Does It Develop?

By Stephen Sawchuk

A key part—though surely not the only part—of early-grades math is ensuring students get the basic arithmetic functions down and, beyond that, making sure they’re able to swiftly and automatically recall addition number combinations and the times tables.

Those skills help kids advance when multidigit arithmetic, fractions, and long division enter the picture—allowing them to focus on more complex problem-solving instead of simple computation.

Nevertheless, a surprising number of misconceptions about fact fluency abound. One common debate—which probably stretches all the way back to the introduction of the abacus, let alone calculators, computers, and Google—concerns whether kids really need to know the multiplication tables when those facts are readily at hand elsewhere. (Hint: They do.)

In a recent Education Week survey of about 300 math educators, most agreed that it’s “essential” for students to have fact fluency in order to work on higher-order, conceptual math problems. But more than a quarter said that it was “helpful, but not essential.”

EdWeek interviewed researchers and reviewed dozens of studies for this explainer. Read on—or jump straight to the bibliography at the end.

Why does math fact fluency matter?

The basic reason why math fact fluency matters, cognitive scientists say, is that it frees up brainpower or working memory to do more complex mathematical work—like figuring out how to structure a multistep word problem, model a solution, or puzzle out systems of equations. It’s harder for students to do those things when they’re simply trying to work through basic arithmetic.

Also, being able to automatically recall math facts seems to be especially important for multiplication: Students have fewer rapid backup strategies to lean on in multiplication if they haven’t stored the times tables in their long-term memory.

For single-digit addition, kids can get pretty fast using strategies like “counting on” from the highest number being added (i.e., 6+5 is counted as “7, 8, 9, 10, 11.”) This becomes impractical in multiplication.

“When you don’t know 6x8, and you’re doing an algebra problem with multiplication, you have to take the time and attention to add 8 and 8 and 8 and 8 and 8 and 8,” said Robert Siegler, a professor of psychology and education at Teachers College, Columbia University. “And, ultimately, you can’t regenerate these forever, as the math gets more complicated.”

Research finds that fluency in these facts is linked with progress in later grades; multiplication in particular is linked to success with fractions, a common tripping-up point for many young students.

How does math fact fluency develop?

There’s a generally accepted understanding that students’ knowledge of the addition number combinations begins by counting up all the digits in a problem and evolves to the more conceptual understanding that whole numbers can be “decomposed,” or broken down or recombined, in a variety of ways. All these stages appear to help students secure the answer to number combinations in their long-term memory.

Many students learn their addition facts without explicit instruction in these strategies, but those with math learning disabilities will need more intensive help. A variety of approaches based in helping kids hone those strategies are promising (see the next heading for details).

In general, there’s much less research on how math fact fluency develops in multiplication. The limited available evidence suggests that there are fewer universal strategies that transfer.

Students generally find 0 and 1 multiplication facts easy because they’re effectively rules-based: Any number times 0 is 0, and any number times 1 is itself. Students tend to skip-count the five’s time tables (5, 10, 15, 20, etc.) And students can use a decomposition strategy for nine’s by remembering their 10’s facts and quickly subtracting 9 from each. Other combinations seem to be somewhat harder to learn and harder for students to store in their long-term memory for instant retrieval.

That is partly why timed exercises are so associated with multiplication: It’s a way of assessing whether students are truly recalling from memory or whether they’re still using the backup strategies.

What are the best ways to develop fact fluency?

Frustratingly, there isn’t a clear answer here. Most of the research derives from interventions for students with math difficulties,
and those studies use a mix of approaches, so it’s hard to identify which element makes the most difference.

In one study on addition facts, for example, researchers compared four different approaches: one was traditional drill work, in which students were shown simple addition and subtraction facts in a computer program and asked to restate them, with a visual aid embedded. In the others, students received explicit teaching beforehand in counting-up or decomposition strategies, in some cases with explicit help practicing and with work on word problems mixed in. All those approaches seemed to be helpful and were linked with improved fluency.

There is far less direct research on multiplication strategies and how they contribute to fact fluency. One study found in comparing two different approaches—one a traditional approach focused on memorization, and a second that integrated some strategies, including number-line work—both seemed to be helpful.

Math anxiety shows up even in young students and can interfere with working memory. Anecdotally, both students and teachers recount feeling stressed when taking timed tests, but it’s less clear that the tests themselves trigger math anxiety or inaccuracy. One study found that removing a timed element improved accuracy on a basic arithmetic test, though another found no difference in accuracy for students with high anxiety on timed vs. untimed exercises.

The difficulty of the math—and whether it’s being graded—also seems to affect matters. A meta-analysis on math anxiety found that it did not interfere with students’ accuracy on simple math problems as much as when problems were more cognitively demanding or related to getting a grade. It also found that these links were higher at the secondary level, rather than at the elementary level.

Some educators point out that if done badly, timed exercises can exacerbate disparities among students.

“In too many classrooms, those ‘mad minute’ type things are creating a dichotomy,” warns Dylan Kane, a 7th grade math teacher in the Lake County district in Leadville, Colo. “The kids who know most of the facts and are developing that automaticity and long-term memory of the facts. [But] the ones who don’t know them are deriving them on their fingers and skip-counting. And if you derive it once and don’t achieve it from memory, you’re not developing it very well.”

A better approach, some teachers say, may be to individualize timed exercises so students are motivated to improve their own time, rather than being compared with one another.
Empower math students with academic language

Ellevation Math is a supplemental program that helps English learners (ELs) understand academic language and key math concepts to better serve their needs. Transform the way that math and EL teachers collaborate to maximize student opportunities with access to:

- Academic language that unlocks rigorous grade-level instruction
- Engaging, real-world scenarios
- Valuable student insights to strategically provide opportunities for math discourse
- Instructional supports like printable reference sheets, read-aloud narrations, and multilingual dictionaries

Learn more about Ellevation Math
Give Cindy Cliche a math word problem, and she can tell you exactly where most students are going to trip up.

Cliche, the district math coordinator in the Murfreesboro City school district in Tennessee, has spent decades teaching elementary schoolers how to tackle their first word problems and now coaches teachers in how to do the same. Kids’ struggles, for the most part, haven’t changed, she said.

Take this problem, which students might work on in 1st grade: There are some bunnies on the grass. Three bunnies hop over, and then there are five total. How many bunnies were there to begin with?

The problem is asking about a change: What’s the starting, unknown quantity of bunnies, if adding 3 to that quantity equals 5? In other words, \( x + 3 = 5 \). But most 1st graders don’t make that connection right away, Cliche said. Instead, they see the numbers 3 and 5, and they add them.

“Nine times out of 10 they’re going to say, ‘eight,’” Cliche said. “They’re number pluckers. They take this number and this number and they add them together or they take them apart.”

This is one of the biggest challenges in word problem-solving, educators and researchers agree—getting students to understand that the written story on the page represents a math story, and that the math story can be translated into an equation.

Making this connection is a key part of early mathematical sense-making. It helps students begin to understand that math isn’t just about numbers on a page, but a way of representing relationships in the world. And it’s one of the ways that kids learn to unite conceptual understanding of problems with the procedures they will need to solve them.

“When students struggle [with word problems], it tends to be everything else they have to do to get to the calculation,” said Brian Bushart, a 4th grade teacher in the West Irondequoit schools in Rochester, N.Y.

There are evidence-backed strategies that teachers can use to help students make these connections, researchers say.

These approaches teach students how to understand “math language,” how to devise a plan of attack for a problem, and how to recognize different problem types. And though they provide students tools and explicit strategies, these techniques are designed to support kids’ sense-making, not circumvent it, said Lynn Fuchs, a research professor in the department of special education at Vanderbilt University.

The goal, she said, is “understanding the full narrative of what’s being presented.”

How word problems are used in early grades

Story problems serve a few different purposes in early grades, said Nicole McNeil, a professor of psychology at the University of Notre Dame who studies students’ cognitive development in math.

They can help connect children’s preexisting knowledge to the math they’re learning in class—“activating that knowledge kids have in their everyday life, and then showing, how do mathematicians represent that?” McNeil said.

Cliche likes to use word problems in this way to introduce the concept of dividing by fractions.

“We’ll tell the kids, ‘I have three sandwich-
creative mathematical thinking, Thompson said. "But we're really driven by standardized tests," she said. "And standardized tests typically have one right answer."

In general, between 30 percent and 50 percent of standardized-test items in math feature these kinds of story problems, said Sarah Powell, an associate professor in the department of special education at the University of Texas at Austin.

"Until things change, and until we write better and different tests, if you want students to show their math knowledge, they have to show that through word problem-solving," Powell said.

**Why students struggle with word problems**

Sometimes, students struggle with word problems because they don’t know where to start.

Just reading the problem can be the first hurdle. If early-elementary schoolers don’t have the reading skills to decode the words, or if they don’t know some of the vocabulary, they’ll struggle, said McNeil.

That can result in students scoring low on these portions of standardized tests, even if they understand the underlying math concepts—something McNeil considers to be a design flaw. “You’re trying to assess math, not reading twice,” she said.

Then, there’s math-specific vocabulary. What do words like “fewer than,” or “the rest,” mean in math language, and how do they prompt different actions depending on their placement in a problem?

Even if students can read the problem, they may struggle to figure out what it’s asking them to do, said Powell. They need to identify relevant information and ignore irrelevant information—including data that may be presented in charts or graphs. Then, they have to choose an operation to use to solve the problem.

Only once students have gone through all these steps do they actually perform a calculation.

Teaching kids how to work through all these setup steps takes time. But it’s time that a lot of schools don’t take, said Cliche, who has also worked previously as a state math trainer for Tennessee. Word problems aren’t often the focus of instruction—rather, they’re seen as a final exercise in transfer after a lot of practice with algorithms, she said.

A second problem: Many schools teach shortcut strategies for deciphering word problems that aren’t effective, Powell said.

Word problem “key words” charts abound on lesson-sharing sites like Teachers Pay Teachers. These graphic organizers are designed to remind students which math words signal different operations. When you see the word “more,” for example, that means add the numbers in the problem.

But unlike key words, schemas don’t tell students what operations to use. Instead, they help students form a mental model of a math event. They still need to read the problem, understand how that story maps onto their mental model, and figure out what information is missing, Fuchs said.

One type of schema, for example, is a “total” or “combine” problem, in which two quantities together make a total: “Jose has five apples. Carlos has two apples. How many apples do they have together?” In this case, students would need to add to get the answer.

But this is also a total problem: “Together, Jose and Carlos have seven apples. If Jose has five apples, how many apples does Carlos have?”

Here, adding the two numbers in the problem would bring students to the wrong answer. They need to understand that seven is the total, five is one part of the total, and there is another, unknown part—and then solve from there.

To introduce schemas, Vanderbilt’s Fuchs said, “we start with a child and the teacher representing the mathematical event in a concrete way.”

Take a “difference” problem, which compares a larger quantity and a smaller quantity for a difference. To demonstrate this, an early-elementary teacher might show the difference in height between two students or

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**Evidence-based strategies for helping struggling students**

So if key words aren’t an effective strategy to support students who struggle, what is?

One evidence-based approach is called schema-based instruction. This approach categorizes problems into different types, depending on the math event portrayed, said Fuchs, who has studied schema-based instruction for more than two decades.

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Take a “difference” problem, which compares a larger quantity and a smaller quantity for a difference. To demonstrate this, an early-elementary teacher might show the difference in height between two students or
the difference in length of two posters in the room.

Eventually, the teacher would introduce other ways of representing this “difference” event, like drawing one smaller and one larger rectangle on a piece of paper. Then, Fuchs said, the teacher would explain the “difference” event with a number sentence—the formula for calculating difference—to connect the conceptual understanding with the procedure. Students would then learn a solution strategy for the schema.

Children can then use their understanding of these different problem types to solve new problems, Fuchs said.

There are other strategies for word-problem-solving, too.

- **Attack strategies.** Several studies have found that giving students a consistent set of steps they can use to approach every problem has positive effects. These attack strategies are different from schemas because they can be used with any problem type, offering more general guidance like reminders to read the problem and pull out relevant information.

- **Embedded vocabulary.** A 2021 study from Fuchs and her colleagues found that math-specific vocabulary instruction helped students get better at word problem-solving. These vocabulary lessons were embedded into schema instruction, and they focused on words that had a specific meaning in a math context—teaching kids the difference between “more than” and “then there were more,” for example.

- **'Numberless’ problems.** Some educators have also developed their own strategies. One of these is what’s called “numberless” word problems. A numberless problem has the same structure as a regular story problem but with the quantities strategically removed. An initial statement might say, for example, “Kevin found some bird feathers in the park. On his way home, he lost some of the feathers.”

  With numberless problems, instead of jumping to the calculation, “the conversation is the goal,” said Bushart, the 4th grade teacher from New York, who has created a website bank of numberless problems that teachers can use.

  “We need more researchers focused on what are the best structures? What order should things go in? What is the appropriate scope and sequence for word problems?”

  NICOLE MCNEIL
  Professor of Psychology, University of Notre Dame

  The teacher talks with students about the change the story shows and what numbers might be reasonable—and not reasonable. The process is a form of scaffolding, Bushart said: a way to get students thinking conceptually about problems from the start.

  **Balancing structure and challenge**

  These approaches all rely on explicit teaching to give students tools that can help them succeed with problems they’re likely to see often in class or on tests.

  But many math educators also use word problems that move beyond these common structures, in an attempt to engage students in creative problem-solving. Figuring out how much structure to provide—and how much challenge—can be a delicate balance.

  These kinds of problems often require that students integrate real-life knowledge, and challenge them to “think beyond straightforward applications of mathematical situations,” said McNeil of Notre Dame.

  There may be an extra number in the problem that kids don’t have to use. Or the problem might pose a question that would lead students to a nonsensical answer if they just used their procedural knowledge. For example: 65 students are going on a field trip. If each bus can hold 10 students, how many buses are needed?

  Students might do the calculation and answer this question with 6.5, but that number doesn’t make sense, said McNeil—you can’t have half a bus.

  In a 2021 study, McNeil and her colleague Patrick Kirkland rewrote some of these challenging questions in a way that encouraged students to think more deeply about the problems. They found that middle school students who worked on these experimental problems were more likely than their peers to engage in deep mathematical thinking. But, they were also less likely to get the problems correct than their peers who did standard word problems.

  Other research, with younger children, has found that teaching students how to transfer their knowledge can help them work through novel problems.

  When students are given only problems that are all structured the same way, even minor changes to that format can prevent them from recognizing problem schemas, said Fuchs.

  “What we found in our line of work is that if you change the way the word problem reads, in only very minor ways, they no longer recognize that, this is a ‘change’ problem, or a ‘difference’ problem,” she said, referencing different problem schemas.

  In the early 2000s, she and her colleagues tested interventions to help students transfer their knowledge to more complex, at times open-ended problems. They found that when children were taught about the notion of transfer, shown examples of different forms of the same problem type, and encouraged to find examples in their own lives, they performed better on novel, multistep problems than their peers who had only received schema instruction.

  The results are an example of how explicit instruction can lay the groundwork for students to be successful with more open-ended problem-solving, Fuchs said.

  Exactly how to sequence this learning—when to lean into structure and when to release students into challenge—is an open question, McNeil said.

  “We need more researchers focused on what are the best structures? What order should things go in? What is the appropriate scope and sequence for word problems?” she said. “We don’t have that information yet.”
Kids Need to Know Their Math Facts. What Schools Can Do to Help

By Stephen Sawchuk

All those long multiplication tables. Timed tests and “mad minutes” of worksheet problem-solving. Fluency drills.

Somehow, getting kids to know their basic math facts continues to be at the heart of some of the loudest disagreements in mathematics education.

Let’s put this old misconception to bed: The cognitive science about math learning indicates that, yes, students do need to develop fluency with their multiplication tables and single-digit addition—sometimes called “number combinations”—and be able to recall them automatically. The main reason why? Having these facts at their fingertips frees up working memory for students to attend to problem-solving, applying procedures to more difficult problems, and other tasks.

The real problem is that research doesn’t point to a clear recipe for how to help students develop their math facts.

Adding to the challenge, some teachers labor under the idea that explicit work on fact fluency isn’t fun for kids, is stressful, or ignores deeper conceptual math. In part that is a product of bad practice—like hours on computer programs or poorly crafted timed exercises—but some of it, some teachers note, is also the product of adult baggage.

“Multiplication facts seem boring to us because we know them,” noted Dylan Kane, a 7th grade math teacher in the Lake County district in Leadville, Colo. “But successfully learning new things is motivating for students. Because it’s old and boring for us doesn’t mean it’s old and boring for them.”

For this story, Education Week examined dozens of studies, spoke to researchers on cognition, and interviewed practicing teachers. We asked them how they approach fact fluency—and how to balance this piece of the math puzzle against all the other things that should happen in the elementary math classroom.

In general, they said, math-fact fluency work can and should be a purposeful part of the math classroom. But it doesn’t have to take forever—or be a drag: Keep the practice relatively short but consistent, keep it well-sequenced, and don’t try to do too much at once, they advised.

“Fluency isn’t fun for kids, is stressful, or ignores deeper conceptual math. In part that is a product of bad practice—like hours on computer programs or poorly crafted timed exercises—but some of it, some teachers note, is also the product of adult baggage.”

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It needs to be a long-term project where students are making gradual, incremental, but steady progress that they can see.”

DYLAN KANE
7th Grade Math Teacher,
Leadville, CO

1. Focus on student mastery of a few facts at a time—and space practice out

Whether students are building their fact fluency with flashcards, worksheets, dice games, or other common tools, there’s a tendency to try to do too much at once, cautions Nicole McNeil, a professor of psychology and the director of lab on cognition, learning, and development at the University of Notre Dame.

“When you use all the number combinations randomly, that is too much for a kid to remember. You have to structure it in the way we know works for retrieval practice—three facts, maybe four, maybe two,” she said. “You can’t become fluent with all of them at once.”

One teaching approach here is called incremental rehearsal: Teachers present a new math-fact flashcard alongside one that a student already knows. As the teacher quizzes the students, the teacher gradually adds more known facts, increasing the time between the known facts and the new one the student is learning.

Many online games purport to help with fact fluency, but as usual, the devil is in the details, and teachers need to know precisely how these programs work, said Kane, the Colorado teacher. He likes an online computer program he uses with his middle school students who still struggle with math facts. It has a limited number of flashcards, and if a student gets a math fact wrong, the computer adds it back into the pile for the next day’s work. (The program only allows kids to go through the cards once a day.) It also starts with smaller numbers, gradually adding more difficult ones as kids’ fluency develops.

In effect, the program handles the spaced repetition element that seems to help kids remember their math facts. “That increases the probability that getting the fact right yesterday will help them retrieve it successfully today,” he said.

In all, Kane said, teachers also need to be aware that fact fluency takes time to develop and isn’t one-and-done.

“I think in too many places, this is seen as ‘one unit,’ and there isn’t enough regular, steady, low-dose practice throughout the year,” he said. “It needs to be a long-term project where students are making gradual, incremental, but steady progress that they can see.”

2. Teach strategies—but know that they have limitations

Many students will ultimately intuit strategies that help them learn their addition facts. But explicit teaching of these backup strategies can be helpful—like “counting on,” in which students count up from one of the numbers to be added (i.e., 6+5 is counted out as “7, 8, 9, 10, 11”). Decomposition, in which students learn that they can break larger numbers down to smaller ones to make it easier to add, can also help kids who struggle.

This is probably because it helps students to
How much instructional time do you spend working on fluency with math facts (e.g., single-digit addition, multiplication, times-tables)?

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*Results show responses from math educators. SOURCE: EdWeek Research Center survey, April 2023

unite conceptual knowledge about how whole numbers work with the facts themselves.

In addition to regular retrieval practice, “I do think it is also helpful to give kids some of these backup strategies,” said Notre Dame’s McNeil. “Activating existing knowledge is part of what helps it stick when you’re trying to use retrieval practice, because you can anchor it to some of your existing long-term memory.”

Multiplication can be trickier because fewer strategies seem to transfer across different number sets. The strategies also take longer than in addition, and it’s not as clear whether teaching strategies explicitly helps students reach automatic recall.

Doubling is one of the strategies that Brian Bushart, a 4th grade teacher in the West Irondequoit Central school district in New York state, teaches explicitly. As early as 1st grade, students learn doubles like 2+2, 3+3, 4+4, and so on—essentially, the two’s times table. They can build on that knowledge when moving into multiplication.

“You can ask students, what if you double again? And you make the connection that if you’re doubling again, you’re multiplying by four,” he said. “You’re showing students that you don’t need to have a bunch of random skills, because doubling ends up working for a chunk of your multiplication facts.”

Once students learn 3x3 is 9, then they can also make the connection that double that, 6x3, is the same as 2x9 and makes 18, and so on.

“These aren’t just cute strategies, they’re actually the basis of multiplicative thinking, which is a huge goal we want for students, because they come to us in 3rd and 4th grade with additive thinking,” Bushart said.

3. Keep broader goals about math in mind

Some of the historical drama over math facts grows out of the slipperiness of the term “fluency,” which in the literature sometimes refers just to math facts and, in other cases, includes learning the standard algorithms for problem-solving (like regrouping or “carrying” in multidigit addition). For many teachers, like Jill Milton, an elementary math specialist with the Duxbury school district in Duxbury, Mass., fluency also means an overall kind of flexibility with numbers.

“If I have a student who is solving 9+5, I want them to have the flexibility to say, ‘Well, I can easily do some compensation and look at that as 10+4,’” she said. “It gives students this step-in to say, ‘I can manipulate these numbers and have the control over how I answer the problem.’”

The goal of automaticity, procedural knowledge, and conceptual knowledge of math are often set up in math education literature as opposing interests, but research suggests that they actually develop together.

“We want these facts encoded in long-term memory, but we want them as networks of connected ideas, flexible representations that we can draw on in different ways,” Kane, the Colorado teacher, said.

One way to do that is to include number lines and other representations as part of work on math facts, said Daniel Ansari, a professor of psychology and the Canada research chair in developmental cognitive neuroscience at the University of Western Ontario in London, Ontario.

“The most useful way I like to think of it is from concrete representational to abstract,” he said. “From manipulatives, then you start to do things around number lines and, at the same time, also develop thinking about solving problems in multiple ways and comparing problem-solving strategies. Just practicing question-and-answer alone is not going to do it.”

And when working on fact fluency through traditional activities aimed at fact retrieval, teachers can help shore up students’ knowledge of undermining mathematical concepts. Take flashcards, for instance. McNeil says that she always includes the equal sign on the cards she uses for tutoring, so they read 2+5 = _ or _ = 2+5.

Her research indicates that many students struggle with the concept of mathematical equivalence and the importance of the equal sign, which is linked to later success in algebraic thinking.

Similarly, when Bushart does flashcards with students, he lists two facts for each flashcard: 6x7 and 7x6 are really the same math fact, not two unrelated ones. That reinforces the commutative property of mathematics—that number order does not matter for addition and multiplication. (It does for subtraction and division.)

Milton, meanwhile, begins many of her lessons with a number talk. Number talks are a common practice in which a teacher puts a question up on the board and has students narrate out loud which strategy they used to solve it, observing and commenting on the different approaches.

There is little empirical research on number talks, and some researchers like McNeil doubt they help students store math facts in memory. But they likely do help improve students’ making sense of numbers, exploring different procedures for solving problems, flexibly selecting among strategies, and math reasoning more generally, she said.
4. Structure timed exercises carefully

For many educators, the idea of timed testing immediately conjures up images of sweaty palms from their own school days. These exercises are included in several popular curriculum series, and many educators remain deeply skeptical about their merits.

Sometimes known as “mad minutes,” the exercise requires students to solve a specified number of basic problems in that amount of time. Many teachers—and some scholars—contend such activities fuel anxiety among kids, despite a lack of conclusive research.

Milton is among them: She finds timed testing “lazy.” Plus, she said, the exercises don’t produce much actionable information to determine which strategies students could bolster: “If it’s not one that’s focused on giving me results, it’s not useful.”

But there is a benefit to timed exercises, other educators counter: The drills require students to retrieve math facts from memory rather than falling back on the strategies.

“We have to practice what we want to become fluent with. There is no amount of practice with backup strategy in multiplication that is going to get a student to produce a product in three seconds or less,” said McNeil.

The proponents say the way to gain a benefit out of timed exercises without all the bad vibes is to reduce the pressure. It should focus on fewer facts at a time, and students should try to beat their own time in solving a select sample of facts—not get compared with their peers on a poster board hanging on the wall for all to see.

“The way I introduce it is to ask them to solve as many as they can, then I will sort of go over the answers again orally before the second chance, and then we time it again, and they love beating their time,” said McNeil, who also tutors in a local school district.

“They’re getting so much experience of meeting a goal, and feeling good when they do it, they ask for it at each session.”

Bushart, similarly, tailors students’ timed exercise to their own progress. He also says it helps to explain to students why he’s requiring them to do the timed drills.

“When they do timed tasks, I tell them: ‘This is how your brain works. I know you know how to skip count. I know you know how to double. But when you want to remember, you have to practice remembering it, and I have to put you in a situation where you don’t have a lot of time to do that extra stuff,’” he said.

Which of the following best describes the way you teach fluency with math facts?

- I expect students to memorize them, and we explicitly practice them using drills and timed tests
- I expect students to memorize them, but I assign practice/assessment infrequently
- I teach them different methods for basic processes (e.g., addition, subtraction, multiplication) which they practice
- I do not teach fluency with math facts

*Results show responses from math educators.

SOURCE: EdWeek Research Center survey, April 2023
Ensuring equitable math education for multilingual learners (MLLs) means providing students with access to rigorous content and rich language support so they can fully participate in the classroom.

A student who is still developing English language skills may be hesitant to speak up in a classroom setting and miss a key learning opportunity. When learners have access to rigorous content combined with confidence-building supports like Ellevation Math Primers, they are provided with low-stakes opportunities to develop their comprehension and expression skills. Our program drives equitable access to rich content and deep learning experiences that align with evolving state standards.

How can we improve academic language support?

<table>
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<tr>
<th>Instead of this</th>
<th>Try this</th>
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<tbody>
<tr>
<td>Pre-teaching vocabulary with a list of words and definitions</td>
<td>Introduce new language in content-related materials. Try using a video, an image (like a meme or an infographic), or audio to introduce vocabulary with meaningful context.</td>
</tr>
<tr>
<td>Vocabulary quizzes as the first chance for students to demonstrate understanding of academic language</td>
<td>Build in opportunities for students to use words before being tested on them. Make more space for discussions and set targets for what language you expect to hear, and for what purpose.</td>
</tr>
<tr>
<td>Relying on students to find the meaning of words in a dictionary without providing meaning or context</td>
<td>Create a language-rich classroom with the academic language that students are expected to use visible and accessible at all times, i.e.: posted on anchor charts with images and supporting context. This will empower students to seek out and find a word for themselves and bring it into their learning.</td>
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It can be tempting to see math class as a place to only solve problems on a whiteboard, but English learners need more agency to drive mathematical talk and idea-building. Here are 4 actionable ways to foster EL student discourse in your math classroom:

1. **Build confidence with conversation preparation and structured interaction**
   Include activities like pair-shares to help students push themselves to clarify problems, not just answer them. Preparation activities, like Confidence Questions, give students opportunities for written practice explaining and arguing math ideas.

2. **Enhance learning with real-world application**
   Use real-world scenarios to help students build background knowledge so that they can practice, experiment, and communicate how math works. Emphasize clarifying terms and justifying ideas when talking to fill information gaps.

3. **Formatively assess students’ evolving ideas**
   Go beyond traditional assessment by using a tool like the Ellevation Math data dashboard. By taking a more holistic view, math and EL teachers can work together to develop their student’s academic language and content understanding.

4. **Empower students to drive their own learning**
   Give students agency to defend their own math learning and communicate it independently. Using data to inform instruction, teachers can strategically provide opportunities for students to share their ideas.

Benefits of rich math talk include an increased sense of belonging, fostering student agency, and building relationships.
Teachers at Boone County Schools used Ellevation Math to help multilingual learners with academic language and key math concepts. They strategically used valuable student data from formative assessments to determine the best route for intervention and saw a 10% increase in MLL students meeting or exceeding state standards.

“Our MLL teachers are collaborative in the math classrooms. So, when we look at our data after an Ellevation Math Primer has been assigned, if there is a gap in vocabulary, the MLL teacher jumps in and does some intervention work, or small group work. If it is a math test question that we’ve seen a lot of gaps on, then the math teacher does reteaching. So, how it’s impacted collaboration between the teachers has been tremendous.”

- Dr. Geniene Piché, Director of Language Learners, Boone County School District

By providing educators with a way to integrate language and math learning research, the district enabled better collaboration between their math and EL teachers.

With Ellevation Math, teachers have:

- Tools to make high-quality instruction more accessible to multilingual learners
- Data to plan for early intervention
- Personalized support to help students succeed at grade level and beyond

Read the full case study
Improving Math Fluency

Published December 10, 2021

These Early Math Supports Translated to Gains Later On for Vulnerable Students

By Sarah D. Sparks

Immediate academic gains from early-childhood programs often fade as children move into upper elementary school. But a new study suggests math supports in the earliest grades may build on each other over years to create longer-term benefits in math achievement and attendance.

In the latest report of an ongoing evaluation of the Making Pre-K Count and High 5s math programs in New York City schools, the research group MDRC found neither program on its own led to significant, sustained math gains by 3rd grade. Yet students who participated in both the preschool and the kindergarten interventions performed significantly better in math and were less likely to miss school by grade 3, compared with students who did not participate.

In the study, students in 2013-14 were randomly assigned within their public schools to participate in standard preschool and kindergarten or the preschool math curriculum alone or with High 5s, a kindergarten enrichment program in which math tutors each met with three to four children for 30-minute math “clubs” three times a week, either during a free period or outside of school.

The enrichment sessions focused on games, songs, and other activities to help students practice geometry, pattern recognition, and other math concepts, rather than basic counting drills in the standard classes.

Shira Mattera, the study author, said the results suggest “the effects seem to be particularly pronounced for children with the most room to grow.”

Students who started preschool with lower-than-average language and attention skills showed math gains by 3rd grade equal to a quarter to a third of a standard deviation. Two years of math enrichment in preschool and kindergarten were enough to produce 3rd-grade math gains large enough to close about 40 percent of the math performance gap between low- and high-income 4th graders.

Attendance improved for participants

Moreover, the researchers found that 28 percent of the students who participated in two years of math enrichment were chronically absent—meaning they missed 10 percent or more of school days—in grade 3, compared with 37 percent of 3rd graders who had not received the early math services.

While chronic absenteeism in early grades often comes from family challenges and stressors that the study did not measure, the researchers did offer some speculation about why students who received math enrichment missed less school later on, Mattera said.

“In kindergarten, in our earlier years of analysis, we found effects of the programs on children’s attitudes towards math. Perhaps, how kids either viewed school or viewed math might have influenced whether they wanted to come to school,” she said. “It’s also possible that teachers saw children who were doing better in math or liked math better as more engaged,” and so built better relationships with them.

These academic and attendance benefits may be particularly important for school districts working to catch up large cohorts of students who are now entering preschool with less formal early-childhood education as a result of the pandemic.

“Both of these programs build on learning trajectory development. All children develop math abilities on a general developmental sequence,” she said. “If you understand the skills that are associated with that sequence, you identify activities that are appropriate for meeting the child’s needs and learning goals to move them to the next part of the sequence.

“I think it makes a nice addition to how people can think about the wide range of skills that children will be coming in with after COVID, because while some students may be coming in with lower skills, because they’ve had missed opportunities, some children may have had a different set of opportunities and teachers are really going have to differentiate across that,” she said. “And this gives an entre into thinking about how you could use similar math activities to meet the needs of a wide range of children’s skills and experiences.”

—E+/Getty

SHIRA MATTERA
MDRC Study Author

How kids either viewed school or viewed math might have influenced whether they wanted to come to school.”

Attendance improved for participants

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Algebra 1 is a Turning Point. Here’s How To Help Incoming Students

By Sarah Schwartz

Throughout the pandemic, data from testing has shown that students are struggling in math, making less progress than they might have in other years. Teachers, too, have said that routines core to their instruction are much harder to do with virtual learners—like showing lots of visual representations, working out problems collaboratively, and having structured student discussions about math concepts. Even with screensharing and digital math tools, they say, it’s not quite possible to recreate the kind of classroom setting where students can work with manipulatives, groups can collaborate on whiteboards, and teachers can evaluate understanding in real time.

Students in all grades may require extra math support next year, but experts say this need is especially urgent in Algebra 1. The course is often the first math class taken in high school, and it’s a gatekeeper for students to ask each other for help or bounce ideas off of one another.

On average, a signifier of whether they will graduate on time or not. Passing Algebra 1 is a graduation requirement in most states.

For this story, Education Week spoke with a dozen instructional experts, teachers, parents, and students about what students starting Algebra 1 next year need and how schools can support them. Representing their reflections and insights is “James,” a composite student about to enter high school and start Algebra 1.

Where things stand for James

When the pandemic hit, James was in the spring of his 7th grade year. Math wasn’t his favorite subject, but there were parts of it he found satisfying—like the moment when an understanding finally clicked into place after lots of examples and repetition. And he liked that math was a social subject, a class where it was not only allowed but encouraged for students to ask each other for help or bounce ideas off of one another.

He spent most of 8th grade in remote learning. Staring at the screen all day was hard. He would get headaches, and his phone was a constant distraction. Sometimes the Zoom feed would lag and he would miss parts of the math notes his teacher gave. The class moved a lot faster online than he was used to, with fewer opportunities to see his teacher work out example problems.

Asking questions was a drawn-out process. He’d have to stop the assignment, email the teacher, and wait for a response before he could keep going. If he were in a physical class, he might have turned to one of his peers for help. But it felt uncomfortable to do that online, when he didn’t know most of his classmates that well.

Instead, he relied on math websites where he could plug in a problem or an equation and get the answer. He wasn’t failing, but he felt like he was barely keeping his head above water.

James’ school opened up for in-person in March 2021. When he came back to the building, his teacher quickly realized that he was struggling with a lot of skills she thought he had mastered—skills he would need to be successful in 9th grade, in Algebra 1. For example: At home, he’d relied on online tools to graph linear equations for him. He wasn’t sure how to do it by hand. And he struggled when asked to find all the positive and negative factor pairs for a number.

But he also had some deeper misunderstandings and unfinished learning around number sense. He was still a bit shaky with fractions and decimals: He might measure 7 inches on a ruler and note the value as 0.7 feet, rather than 7/12 of a foot. Presented with an equation like \( \frac{1}{2}x + 3 = 7 \), he knew to subtract 3 from both sides. But then he wasn’t always sure how to “undo” the fraction.

In a normal 8th grade year, teachers said, they would take every opportunity to correct those misunderstandings in the moment and shore up students’ comfort—not only with fractions and decimals, but exponents, radicals, and negative integers, too. Wendy Habeeb, an 8th grade math teacher at Salida Middle School in California, said that she is constantly plotting on a number line on the white board, so that students can see connections between different expressions of numbers—that the square root of 64 is 8, which is the same as \( \frac{16}{2} \), for example.

“Having that ability to see relationships between numbers is what leads to success in Algebra 1,” said Phil Murray, a high school math teacher at Early College Opportunities High School in Santa Fe, N.M.

But online, it was harder for teachers to do that kind of constant reinforcement, and harder for students to internalize it. Now that James is back in the classroom, asked to explain his thinking, he draws a blank. He’s hesitant to volunteer answers because he’s afraid they’ll be wrong, and he doesn’t want to look like he’s farther behind than everyone else.
He’s nervous about starting Algebra 1. He’s already having a hard time keeping all the numbers and letters straight in his head, and he knows it’s only going to get more complicated from here. Next year also means the start of high school: a new group of students, new teachers, and the expectation, he worries, that he’ll be able to handle more advanced work on his own. He doesn’t feel ready.

What algebra teachers can do

Even in a regular year, teachers say, students come into Algebra with varying degrees of readiness. But this year, the range might be even greater, depending on what opportunities and resources they had during remote learning. James has trouble with fractions, while another student might be fine with fractions but struggle with exponents. For that reason, teachers and experts recommend, lessons should start with checks for understanding.

Teachers can figure out what skills and understandings are prerequisites for the new concept they’re starting to introduce, and then give students a couple of questions that would allow them to show their knowledge—or demonstrate that they have unfinished learning. Then, teachers can develop a task or mini-lesson to shore up that prerequisite skill, and make explicit its connection to the new learning. For example, teachers could review the basics of linear functions and how to plot them on a graph right before introducing slope-intercept form.

Experts recommend this kind of targeted, just-in-time support instead of remediation (having James repeat entire units from 8th grade math before moving on to Algebra 1 content). Remediation can be demotivating, said Amy Getz, the interim director of K-12 education strategy, policy, and services at the University of Texas at Austin’s Charles A. Dana Center. It also can push students who are struggling further behind, by limiting their access to grade-level content, she said.

James’ teacher can show multiple representations for new concepts, something that James always found helpful in the classroom—for example, drawing explicit connections between the way a linear function looks written as a mathematical expression, the way it looks as a graph, and real-world examples James might encounter. Teachers can also be explicit about the connections between word problems and the equations meant to solve them, teaching solution methods for different types of problems.

And the number line that Habeeb, the California middle school teacher, uses doesn’t have to stay in 8th grade: Algebra teachers can continue to plot radicals, exponents, and fractions if students are having a hard time conceptualizing their magnitude. For example: The idea that the square root of 16 is the same as 4 is the same as 2 squared can feel really abstract to students, said Sheng Lor, another 8th grade math teacher who works with Habeeb. But when she plots numbers like these at the same point on a number line, she said, “it was a like a switch in their head.”

Next comes practice, practice, practice, teachers say—opportunities to build fluency and confidence that students might not have had while learning remotely. Group practice, specifically, also allows teachers to listen to students’ thought processes. James’ teacher could listen in to his group conversation—asking guiding questions to explore his thinking, reinforcing his use of mathematical language, and addressing any misunderstandings in the moment.

But teachers will also have to get students comfortable having these kinds of group discussions again—important for students in all grades, but crucial for incoming 9th graders who may not know their classmates. Lor said that comfort level doesn’t just happen. She had to intentionally set aside time for students to develop relationships. This spring, she had some additional time with her math students due to state testing schedules. She chose to spend part of it just talking—having students share what they were doing over the weekends, for example.

It was a tough choice, deciding to chat instead of squeezing in one more math problem, because Lor knew that these students had already missed so much learning time. But it paid off: Her students were quicker to participate in turn-and-talks during the short time left in the school year.

High school math teachers—who might not usually spend as much time outlining classroom norms as their middle school counterparts—could spend more time on that this year, Getz said. “You model for the students how you can ask questions to try to understand someone’s reasoning, making it really clear that getting a wrong answer can sometimes be a really important step in the learning process.”

Insights for all teachers

Teachers, experts, parents, and students focused on two big takeaways. First, in math, all of this focus on relationship building and social-emotional learning isn’t an extra, teachers say. It’s integral to students’ academic success.

If students don’t feel comfortable saying they don’t understand, if they aren’t willing to tackle a challenging problem or share their ideas in a group, then they won’t be able to get the practice they need to achieve fluency, or ask the questions that can lead to deep conceptual understanding.

“All the time I would [have liked] to ask a question, but I was afraid of what was going to happen.”

CAMRYNN SMITH
9th Grade Smith
Salida, CA

“Be patient,” Smith advised teachers. “Sometimes it’s really hard getting back into the groove of things.”

But getting back into the groove doesn’t have to mean easing off the challenge. Which leads to the second point: Give all students access to grade-level content. Helping students master challenging work with appropriate support keeps them on track, so that they’re prepared for higher level math and can succeed in the courses they need for graduation. And it can also build their confidence.

“I’m waiting for that ‘aha’ moment when she’s actually excited about the fact that she’s getting it,” said Christina Laster, a Palm Springs, Calif., parent of a rising 10th grader who was in Algebra 1 this past year. “I hope that it’s not as emotionally draining.”

CAMRYNN SMITH
9th Grade Student,
Salida, CA

Improving Math Fluency

Education Week • SPOTLIGHT
Math Trauma Is Real. Here’s How You Can Prevent It
Why too many students end up deciding, “This math stuff isn’t for me”

By Viveka Vaughn

Toward the end of last school year, I sat on a virtual mathematics panel discussing the resilience of students in the face of COVID-19’s traumatic educational consequences. I began to examine the social-emotional implications of the pandemic in a field I have been teaching in for over 20 years: mathematics.

The pandemic exacerbated the inequities of educational resources, leaving many students, especially those from high-needs districts, behind mathematically. These widening disparities are particularly damaging for students already at risk for “checking out” of math because of hostile classroom experiences.

Whenever I inform people of my occupation, they are animated with a look of joy or misery as they remember their feelings for math—and it’s usually the latter. They often regale me with stories of negative classroom experiences or encounters, usually involving a teacher embarrassing or ridiculing them in class.

“I writhed like a snake over coals if it came near me,” writes poet Chase Twichell in the poem about “Math Trauma.” This sentiment is echoed in many of the anecdotal stories about people’s math experiences.

One person vividly remembered trying to gain clarity on a math concept and the teacher replying, in front of the class, “That was a dumb question.” They never asked a question in a math class again. For many, that one experience became so traumatic that it led them to ignore math and any of its adjacent fields of study.

For me, math trauma is an event (or a sequence of events) where an educator or another person of authority chooses to embarrass, scorn, or deride students for their mistakes rather than celebrate their courage. That math trauma is then triggered whenever they encounter a math problem or math conversation for the duration of their academic years. Even outside of school, they refuse to recognize daily habits such as balancing a checkbook, counting money, or estimating prices as practicing math.

As an undergraduate, an academic advisor told me that I would never complete a math degree, let alone get a Ph.D. in anything. That, for me, was a traumatic moment. It was commonplace for people to underestimate my abilities based on my race, gender, and public school background.

Instead of letting those negative comments and assumptions dominate and traumatize me, they became my motivation and launching pad for success. I always remembered the teachers I had in middle and high school who reinforced courage, leadership, and discipline.

As a first-generation college student, I took inspiration from the television show “A Different World” as a positive image of a Black college experience. In one episode, a professor made her class stand up and recite the chant, “You are a voice in this world and you deserve to be heard.” As I faced discrimination in math, this mantra continued to resonate with me. I also had a community of family, friends, and mentors that helped me counter those negative stereotypes and prevent math trauma from taking root.

However, many students who don’t receive such encouragement and support end up reacting to negative experiences in the classroom by concluding, “This math stuff isn’t for me.”

Math trauma may be amplified for underrepresented groups in math. As a field historically dominated by white men, mathematics is rife with norms that have excluded women and students of color. Stereotype threat and teachers’ preconceived notions of students’ math abilities can further hinder minoritized groups. If we want to diversify the math arena, alleviating math trauma and encouraging math curiosity from all races and gender is necessary.

Why focus on this issue now? As educators recovering from the pandemic, we are channeling our creativity, innovation, and imagination through our pedagogy and curriculum. Let us extend this same gift to our
students, especially in math. Now is our chance to reset, recalibrate, and create new norms in countering the previous standards that have been a disservice to our students.

We must eliminate bias and assumptions about our students and permit time for them to process, develop, and construct math abilities, comprehension, and reasoning. As we plan novel ways to teach in this new normal, let us move forward with a renewed zeal to leave outdated standards in the past.

For educators, especially those teaching students of color, I caution you to be careful on the assumptions, biases, and predispositions you bring to the classroom. Focus on supporting and encouraging young minds in math class and appreciating their ingenuity.

My teaching goal is to convey a culture of acceptance and appreciation of differentiated math abilities. At the beginning of each semester, I introduce my students to psychologist Carol Dweck’s concept of fixed mindsets vs. growth mindsets. We focus on why it’s important to understand how the brain grows and how learning is malleable and not static. However, I wonder if we educators have more of a fixed perspective when it comes to believing in our students.

For the past 12 years, I have attended an improvisation class, where I learned a valuable technique that I now incorporate into my math instruction: the “Yes, and...” principle. With this approach of accepting and expanding on whatever was just said, no question is ever unimportant. With each question that a student asks, we can recalibrate. This creates a teachable moment and the chance to revise my strategy to help students find the correct answer. Many times, I encourage students to explain their reasoning of a problem without providing a final answer. This encourages students to see themselves as math thinkers.

Let us not be the reason students feel deterred or traumatized from pursuing math or another STEM subject. We must encourage and acknowledge our students’ strengths whenever we can. We can also provide all our students with the necessary tools to become effective and proficient in math. Once we institutionalize these strategies in our classrooms, then perhaps the next time you tell someone you teach math, you’ll get a smile of delight.

Viveka Vaughn is an associate professor in the mathematics department at Spelman College. Her research focuses on equity issues in mathematics.